MATH 2130 LINEAR ALGEBRA
HOMEWORK 10
DUE 2025 NOVEMBER 2

Problem 1 (P5)

Show that the function $f\colon \mathbb{R}^2 \to \mathbb{R}^3$ given by

$$f(x, y) = (2x + y, 3x + y, x - y)$$

is a homomorphism.

Problem 2 (P5)

Show that the function $g\colon \mathcal{P}_2 \to \mathrm{Mat}_{3\times 1}$ given by

$$g(ax^2 + bx + c) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

is a homomorphism.

Problem 3 (P5)

Show that the function $h\colon \mathcal{P}_2 \to \mathbb{R}^3$ given by

$$h(ax^2 + bx + c) = (a, a, b + c)$$

is a homomorphism.

Problem 4 (P5)

Show that the function $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by

$$T(x, y, z) = (x + y, 2x + 3z, 0)$$

is a homomorphism.

Problem 5 (P5)

Show that the function $\alpha\colon \mathbb{R}^3 \to \mathbb{R}$ given by

$$\alpha(x, y, z) = 7x + 2y - 3z$$

is a homomorphism.