

MATH 2130 LINEAR ALGEBRA
HOMEWORK 11
DUE 2025 NOVEMBER 9

PROBLEM 1 (P6)

Let $\alpha: \mathbb{R}^2 \rightarrow \mathbb{R}$ be given by

$$\alpha(x, y) = 4x - 12y.$$

Find a basis for $\text{Ker}(\alpha)$.

PROBLEM 2 (P6)

Let $T: \mathbb{R}^3 \rightarrow \text{Mat}_{2 \times 2}$ be given by

$$T(a, b, c) = \begin{bmatrix} a + 2b & a + 3c \\ 2b - 3c & 2a + 4b \end{bmatrix}.$$

Find a basis for $\text{Ker}(T)$.

PROBLEM 3 (P6)

Let $\psi: \mathbb{R}^3 \rightarrow \mathcal{P}_2$ be given by

$$\psi(a, b, c) = ax^2 + (a + b + c)x + (2b + 2c).$$

Find a basis for $\text{Ker}(\psi)$.

PROBLEM 4 (P6)

Let $f: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be given by

$$f(x, y, z) = (5x - 2y, 3x + y + z, 2x).$$

Find a basis for $\text{Im}(f)$.

PROBLEM 5 (P6)

Let $g: \text{Mat}_{2 \times 2} \rightarrow \mathbb{R}^2$ be given by

$$g\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a - 2b + c - d, 3a - 6b + 3c - 3d).$$

Find a basis for $\text{Im}(g)$.