MATH 2130 LINEAR ALGEBRA
WEEK 9 QUIZ
2025 OCTOBER 24

Problem P1-1

For which values of k are there no solutions, many solutions, or a unique solution to the system

$$x - 3y = k + 2$$

and

$$4x - 12y = k?$$

Problem P1-2

Use Gauss's method to find the unique solution to the system

$$3x + 4y + z = 1,$$

$$2y + z = 2,$$

and

$$x + 2y + z = 3.$$

Problem P2-1

Find the reduced echelon form of the matrix

$$\begin{bmatrix} 6 & 2 & 1 & 1 \\ -2 & 1 & 0 & 3 \\ 2 & 4 & 1 & 7 \end{bmatrix}.$$

Problem P2-2

Use Gauss-Jordan reduction to solve the system

$$4x_1 + 3x_2 + x_3 = 0,$$

$$3x_1 - x_2 + 8x_3 = 5,$$

and

$$3x_1 + x_2 + x_3 = 0.$$

Problem P3-1

Show that

$$\left\{ (x, y, z) \in \mathbb{R}^3 \;\middle|\; 5x - y + 7z = 0 \right\}$$

is closed under scalar multiplication.

Problem P3-2

Show that

$$\left\{ \begin{bmatrix} a & -b \\ b^2 & a \end{bmatrix} \;\middle|\; a, b \in \mathbb{R} \right\}$$

is not a vector space under the usual matrix operations.

Problem P4-1

Show that $\{(0,0,3),(0,2,2),(1,1,1)\}$ is a basis for $\mathbb{R}^3$.

Problem P4-2

Show that $\left\{x^2 - x, 3x^2 - x, 4x + 2\right\}$ is a basis for $\mathcal{P}_2$.

Problem P5-1

Is the function $f\colon \mathrm{Mat}_{2\times 2} \to \mathbb{R}^4$ given by

$$f\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a - c, a + b - c, b - c, a + c)$$

an isomorphism? Either prove that it is or show that one of the conditions fails.

Problem P5-2

Is the function $h\colon \mathbb{R}^2 \to \mathbb{R}^2$ given by $h(a,b) = (2a + 3b, 3a + b)$ an isomorphism? Either prove that it is or show that one of the conditions fails.

Problem S1

Describe the set of points on the plane through $(2,3,0,0)$, $(1,0,0,1)$, and $(1,4,1,0)$ in $\mathbb{R}^4$. Does the origin lie on this plane?

Problem S2

Find the angle between the vectors $(2,3,0,0)$ and $(1,4,1,0)$ in $\mathbb{R}^4$.

Problem S3

Use the Subspace Test to show that

$$V = \left\{ (x,y,z) \in \mathbb{R}^3 \;\middle|\; 6x + 2y + 3z = 0 \right\}$$

is a subspace of $\mathbb{R}^3$.

Problem S4

Show that $\{(2,1,4),(2,1,2),(2,3,1)\}$ is a spanning set for $\mathbb{R}^3$.

Problem S5

Show that $\{(1,4,-2),(5,3,1),(1,21,-13)\}$ is linearly dependent in $\mathbb{R}^3$.

Problem S6

Evaluate

$$\begin{bmatrix} 1 & 5 & 2 \\ 3 & 0 & 1 \\ 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

Problem S7

Find the inverse matrix of

$$\begin{bmatrix} 1 & 5 & 2 \\ 3 & 0 & 1 \\ 0 & 2 & 4 \end{bmatrix}.$$