Math 2001
Discrete Mathematics
Week 3
Injective and surjective functions

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Today's topics

1. Injective and surjective functions

Injective and surjective functions

- A function $f\colon A \to B$ is *injective* when $f(a_1) = f(a_2)$ implies that $a_1 = a_2$ for any $a_1, a_2 \in A$.
- That is, a function $f\colon A \to B$ is injective if $f(a_1) \neq f(a_2)$ means that $a_1 \neq a_2$.
- A function $f\colon A \to B$ is *surjective* when for each $b \in B$ there is at least one $a \in A$ such that $f(a) = b$.

Injective and surjective functions

- The function $f\colon \mathbb{N} \to \mathbb{N}$ given by $f(n) = n^2$ is injective but not surjective.
- The function $g\colon \mathbb{R} \to \mathbb{R}$ given by $g(x) = x^3 - x$ is surjective but not injective.
- The function $h\colon \mathbb{R} \to \mathbb{R}$ given by $h(x) = x^3$ is both injective and surjective.
- The function $k\colon \mathbb{R} \to \mathbb{R}$ given by $k(x) = x^2$ is neither injective nor surjective.
- Note that f and k are different functions because their domains (and codomains) are different. This matters when checking for injectivity and surjectivity. The formula or rule for the function does not give enough information on its own.