Math 2001
Discrete Mathematics
Week 3
Images and preimages

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Today's topics

1. Images and preimages

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- Let $f\colon A \to B$ be a function. In addition to considering the value of f at the input $a \in A$ (which we usually write as $f(a)$), it also makes sense to consider $f(X)$ for a subset X of A.
- Given $X \subset A$, we refer to the set

$$f(X) = \{\, f(x) \in B \mid x \in X \,\}$$

  as the *image* of X under f.
- We often refer to $f(A)$ as the *image* of f itself. The set $f(A)$ is also known as the *range* of f. (Note that this is **not** the same as the codomain of f in general.)
- We always have that $f(X) \subset B$ for any $X \subset A$.
- A function is surjective exactly when $f(A) = B$.

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- Given $Y \subset B$, we refer to the set

$$f^{-1}(Y) = \{\, a \in A \mid f(a) \in Y \,\}$$

  as the *preimage* of Y under f.
- We always have that $f^{-1}(Y) \subset A$ for any $Y \subset B$.
- We often write $f^{-1}(b)$ instead of $f^{-1}(\{b\})$ for a single element $b \in B$.
- A function is injective exactly when $\left|f^{-1}(b)\right| \leq 1$ for each $b \in B$.