

2001 DISCRETE MATHEMATICS
HOMEWORK 2
DUE 2025 FEBRUARY 8

PROBLEM 1 (S2, 2 POINTS)

Let $A = \{1, 2, 3, 4\}$ and let $B = \{1, 2, 3, 4, 5\}$. For each of the following functions, determine whether the function is injective, surjective, both, or neither. Show your reasoning.

a. $f: A \rightarrow B$ given by $f(n) = n + 1$

b. $g: A \rightarrow B$ given by $g(n) = 3$

c. $h: B \rightarrow A$ given by

$$h(n) = \begin{cases} n - 1 & \text{when } n \geq 2 \\ 1 & \text{when } n = 1 \end{cases}$$

d. $k: B \rightarrow B$ given by $k(n) = 6 - n$

PROBLEM 2 (S2, 2 POINTS)

For each function in Problem 1, find the image of the function.

PROBLEM 3 (S2, 2 POINTS)

Let $f: A \times A \rightarrow A$ be a function and suppose that f is injective. If A is finite, what are the possible values for $|A|$? Give an example of such a function f when A is an infinite set.

PROBLEM 4 (S3, 2 POINTS)

For each of the following statements, write out both the converse of the statement and the contrapositive of the statement. (You do not need to determine whether the statement is true or false.)

a. If I do my homework, I will get a good grade.

b. If an essay is long, then it is well-written.

PROBLEM 5 (S3, 2 POINTS)

For each of the following propositional formulas, write out the corresponding truth table.

a. $P \vee (P \rightarrow Q)$

b. $P \wedge (P \rightarrow Q)$

c. $(P \wedge Q) \vee (P \wedge R)$

d. $(P \vee Q) \wedge \neg(R \rightarrow P)$